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DISCUSSION OF SURFACE CURVES FOR STEADY NONUNIFORM FLOW

(*Published in October, 1951*)

By Ivan M. Nelidov; J. C. Stevens; Maurice Bishay and H. E. Babbitt; K. C. Wu; J. Pietrkowski; and Robert B. Jansen

HYDRAULICS DIVISION

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Power	48, 55, 56, 69, 71, 88, 96, 103, 106, 109, 110, 117, 118, 120, 129, 130, 133, 134, 135, 139, 141 (Discussion: D-XXIII, D-2, D-3, D-7, D-38, D-40, D-44, D-70, D-71, D-76, D-78, D-79, D-86, D-92, D-96)
Sanitary Engineering	55, 56, 87, 91, 96, 106, 111, 118, 130, 133, 134, 135, 139, 141 (Discussion: D-29, D-37, D-56, D-60, D-70, D-76, D-79, D-80, D-84, D-86, D-87, D-92, D-96)
Soil Mechanics and Foundations	43, 44, 48, 94, 102, 103, 106, 108, 109, 115, 130 (Discussion: D-4, D-XXVIII, D-7, D-43, D-44, D-56, D-75 D-86)
Structural	42, 49, 51, 53, 54, 59, 61, 66, 89, 100, 103, 109, 113, 116, 117, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137 (Discussion: D-51, D-53, D-54, D-59, D-61, D-66, D-72, D-100)
Surveying and Mapping	50, 52, 55, 60, 63, 65, 68, 121, 138 (Discussion: D-60, D-65)
Waterways	41, 44, 45, 50, 56, 57, 70, 71, 96, 107, 112, 113, 115, 120, 123, 130, 135 (Discussion: D-8, D-9, D-19, D-27, D-28, D-56, D-70, D-71, D-78, D-79, D-80)

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DISCUSSION

IVAN M. NELIDOV[4].—In sketching the water-surface curves obtained with the aid of the slopes of water, computed by means of Eq. 12, the author has made the water-slope tangents intersect at the midpoints of the depth increments. This equation may be rewritten as follows:

$$\frac{dy}{dx} = \frac{S_o - S}{1 - Gj} = \frac{\frac{dE}{dx}}{\frac{dE}{dy}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which (conforming to the notation of the paper), S is the friction slope and E is the specific energy, or energy referred to the invert. Eq. 2 can be represented as the Bernouilli equation if the depth y_1 is added to both sides, if the friction slope S is introduced, and if both sides are multiplied by dx; thus,

$$-S\,dx = -S_o\,dx + y_2 - y_1 + \frac{V^2_2}{2g} - \frac{V^2_1}{2g}$$

or

$$y_1 + \frac{V^2_1}{2g} + I = y_2 + \frac{V_2^2}{2g} + h_f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which the subscripts 1 and 2 refer to two sections, distance dx apart; $I = S_o\,dx$ is the difference in elevation between the inverts at section 1 and section 2; and $h_f = S\,dx$ is the friction head lost. Eq. 14 may also be rewritten as,

$$E_1 + I = E_2 + h_f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which E_1 and E_2 are the specific energies at sections 1 and 2, respectively.

The only correct way to construct the water-surface curve is by integration. However, cases where integration can be used are rather limited because the channel is not a prism of uniformly varying cross sections. It may be quite irregular and can change abruptly from section to section.

The approximation proposed by the author involves constructing the water-surface profile by means of tangents connecting two consecutive depths in such a way that these tangents meet at the midpoint of the depth increment. It may be noted, that the standard mathematical procedure for constructing a curve is to mark equal horizontal increments dx, with varying vertical increments dy.

The tangents will thus intersect at the vertical ordinates or, if the ordinates are translated a distance $\frac{1}{2}\,dx$ they will intersect at the midpoint between the ordinates.

NOTE.—This paper by Robert B. Jansen was published in October, 1951, as *Proceedings-Separate No. 96*. The numbering of footnotes, equations, illustrations, and tables in this Separate are a continuation of the consecutive numbering used in the original paper.

[4] Civ. and Structural Engr., San Francisco, Calif.

In example 1, if the ordinates 2.5 and 3.0 are selected, the horizontal distance between them will be: $dx = \frac{0.25}{0.00275} + \frac{0.25}{0.00303} = 173.4$ ft. The same dx, computed by the Bakhmeteff method,[2] and using his notation, is obtained as follows: $y_o = 6.12$ ft; $S_o = 0.001$; the hydraulic exponent $n = 2.98$; σ (the critical slope) $= 0.00244$; $\beta = \frac{S_o}{\sigma} = 0.41$; and

$$dx = \frac{6.12}{0.001}\left[\frac{3.00}{6.12} - \frac{2.50}{6.12} - (1 - 0.41)(0.5059 - 0.4155)\right] = 176.0 \text{ ft}$$

in which the varied flow functions are $0.5059 = B\ (\eta_2)$ and $0.4155 = B\ (\eta_1)$. The curve in this example[5] is type M_3 and the flow is rapid and retarded. To use the Bernoulli equation in determining distance dx without trial, Eq. 15 must be solved, from which:

$$dx = \frac{E_1 - E_2}{S - S_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The numerical values entering Eq. 16 are as follows: $E_1 = 2.50 + 7.25 = 9.75$; $E_2 = 3.00 + 5.02 = 8.02$; $S_o = 0.001$; and $S = 0.01095$—determined on the basis of an average hydraulic radius $R_{av} = 2.155$ and an average velocity $V_{av} = 19.8$ ft per sec, with $n = 0.013$. Then, $dx = \frac{9.75 - 8.02}{0.01095 - 0.001} = \frac{1.73}{0.00995} = 173.6$ ft.

If an average value of the friction slopes computed for the end section is used, the distance dx would differ more from the foregoing values. The following numerical data would be used: $V_1 = 21.6$ ft per sec; $R_1 = 2.00$ ft; $V_2 = 18.0$ ft per sec; and $R_2 = 2.31$ ft. Furthermore, $S_1 = 0.01455$, $S_2 = 0.00815$, and $S_{av} = 0.0117$, so that $dx = \frac{1.73}{0.0117 - 0.001} = 162$ ft. In general, the friction loss should be computed by introducing R_{av} and V_{av} for end sections and determining the friction slope on the basis of their values. This may be proved by taking the general expression for friction loss and comparing the results. Thus, integrating the theoretical friction loss with the aid of the Bakhmeteff formulas[6]:

$$h_{f1} = \int S\,dx = \int \frac{(K_o)^2}{(K)} S_o\,dx = \int \frac{(y_o)^3}{(y)} S_o\,dx$$

$$= S_o\,y_o^3 \int_0^L \frac{dx}{\left(y_1 + \frac{y_2 - y_1}{L}x\right)} = S_o \frac{L}{2} \frac{(y_o)^3}{(y_1)}\left[\frac{y_1}{y_2} + \frac{(y_1)^2}{(y_2)}\right]$$

$$= \text{Constant}\left[\frac{y_1}{y_2} + \frac{(y_1)^2}{(y_2)}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

[2] "Hydraulics of Open Channels," by B. A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932.

[5] *Ibid.*, p. 76.

[6] *Ibid.*, p. 82.

In Eq. 17, $K = A\,C\,\sqrt{R}$ = the Bakhmeteff "conveyance" of a cross section, K_o being the conveyance at the normal depth y_o; and, L is the distance along the line of flow from an initial section.

If friction-loss computation is based on the data referred to the average cross section, the expression for friction loss will be:

$$h_{f2} = \int S\,dx = \int \frac{(K_o)^2}{(K)} S_o\,dx = \int y^3{}_o\, S_o \frac{dx}{y^3{}_{av}} = y^3{}_o\, S_o \int_o^L \frac{dx}{\left(\frac{y_1 + y_2}{2}\right)^3}$$

$$= \left(\frac{y_o}{y_1}\right)^3 \frac{L}{2} S_o \frac{16}{\left[1 + \frac{y_2}{y_1}\right]^3} = \text{Constant} \frac{16}{\left(1 + \frac{y_2}{y_1}\right)^3} \quad \ldots\ldots\ldots (18)$$

If the friction loss is computed by taking the average friction slope, the expression then will be

$$h_{f3} = S_{av}\,L = \frac{S_o L}{2}\left[\left(\frac{y_o}{y_1}\right)^3 + \left(\frac{y_o}{y_2}\right)^3\right] = \frac{S_o L}{2}\left(\frac{y_o}{y_1}\right)^3\left[1 + \left(\frac{y_1}{y_2}\right)^3\right]$$

$$= \text{Constant}\left[1 + \left(\frac{y_1}{y_2}\right)^3\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Table 3 indicates that friction computations by the Bernoulli equation should be based on average area and average hydraulic radius, and not on average friction slope. In general, the method proposed by the author gives results within the desired degree of accuracy—that is, within the ordinary

TABLE 3.—COMPARISON OF FRICTION COMPUTATIONS

Friction loss	Equation	VALUES OF y_2/y_1	
		0.7	1.3
h_{f1}	17	Constant [3.42]	Constant [1.36]
h_{f2}	18	Constant [3.25], or −5%	Constant [1.32], or −3%
h_{f3}	19	Constant [3.93], or +15%	Constant [1.45], or +7%

range of variation of depths. However, with rapidly varying depths, all approximation methods (which includes the author's method and the Bernoulli method when finite distances dx are used) are likely to give incorrect results, the degree of error being established only after integration is performed.

It is important to classify the water-surface curves properly so that the type involved is evident at a glance. In Fig. 4, three major classes are noted, first: flow with $y_o > y_c$ (class A), with $y_o = y_c$ (class B), and with $y_o < y_c$ (class C).

The slope of invert in class A is adverse (class A–I), zero (class A–II), and gentle (class A–III); class B is critical; and in class C it is greater than critical. In any horizontal row the first figure shows the water depth y greater than both

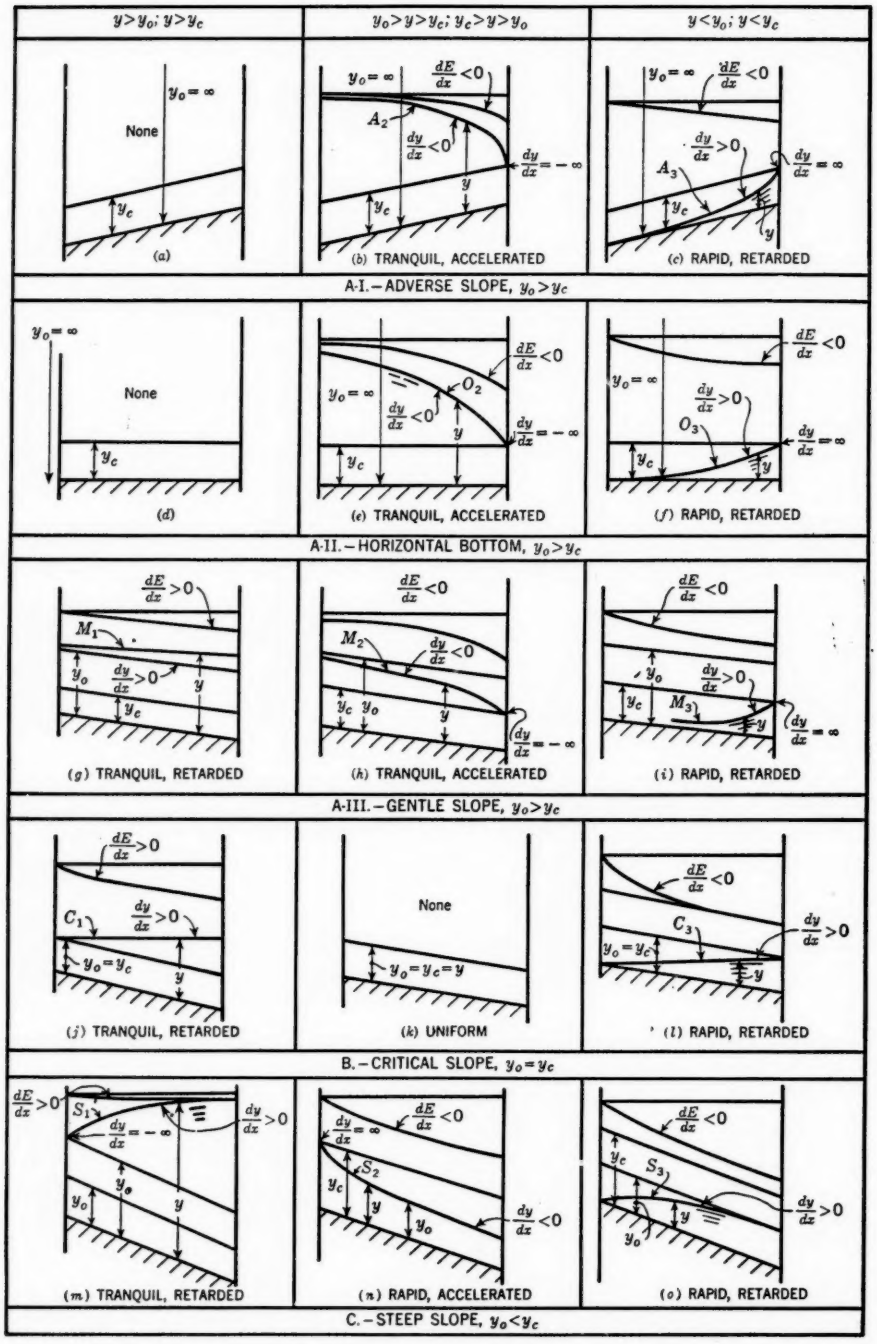

FIG. 4.—CLASSIFICATION OF WATER SURFACE CURVES IN STEADY NONUNIFORM FLOW

y_o and y_c; in the second figure y is between y_o and y_c; and in the third figure y is smaller than both y_o and y_c.

Consistent with the letter symbols adopted for the author, the notation for water-surface curves is the same as that introduced by Mr. Bakhmeteff and the classification itself is only a more graphical presentation of Mr. Bakhmeteff's ideas. As far as the type of the flow is concerned, in the first vertical column the flow is tranquil and retarded; in the second vertical column it is tranquil and accelerated until it becomes flow at critical slope, where it is uniform, and past it, it becomes rapid and accelerated; in the third vertical column the flow, for all cases is rapid and retarded.

The writer has found that this form of classification is helpful in determining the type of a curve involved before beginning the detailed computations.

J. C. STEVENS,[7] M. ASCE.—Engineers still persist in trying to solve a really simple problem in the hardest possible way. Since 1925, the writer has been urging the use of a simple step-by-step method applicable to all cases.[8,9] This method is not original with the writer. It was first presented by Alva G. Husted.[10] It is strictly applicable to any form of water surface profile on sustaining or adverse slopes under sub-critical or super-critical flow on rough or smooth channels provided only they are in uniform channels—that is, rectangular, triangular, trapezoidal, circular, parabolic, elliptical, or otherwise—just so that the area and, consequently, the perimeter, radii, etc., are the same functions of the depth through the reach under consideration.

To illustrate the point that the writer wishes to make it will be necessary to compute the surface curves of Example 2 by the Husted method.

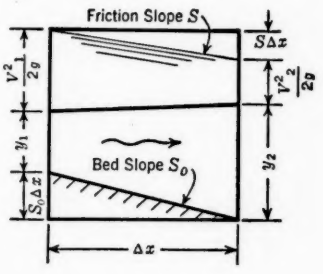

FIG. 5

Fig. 5 illustrates the conditions for this example in which the bed slope is greater than the friction slope, the depth is super-critical and super-normal, and there is a gain in specific energy downstream. From Fig. 5,

$$S_o \Delta x + y_1 + \frac{V^2_1}{2g} = y_2 + \frac{V^2_2}{2g} + S \Delta x \quad \ldots\ldots\ldots\ldots (20a)$$

$$(S_o - S) \Delta x = e_2 - e_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

and

$$\Delta x = \frac{\Delta e}{S_o - S} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20c)$$

in which

$$e = y + \frac{V^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

[7] Stevens & Koon, Cons. Engrs., National Bldg., Portland, Ore.

[8] *Engineering News-Record*, Vol. 95, 1925, p. 550.

[9] *Transactions*, ASCE, Vol. 102, 1937, p. 666.

[10] "New Method of Computing Backwater and Drop-Down Curves," by Alva G. Husted, *Engineering News-Record*, Vol. 92, 1924, p. 719.

the specific energy of 1 lb of water measured as a head above stream bed as datum. Dimension Δx is a finite length of channel over which the surface of the stream is to be considered a straight line; S_o is the bed slope; and S is the friction slope. Table 4 shows the computations for this example. The flow is

TABLE 4.—COMPUTATIONS FOR EXAMPLE 2, BY THE HUSTED METHOD

y = depth; A = area; p = wet perimeter; R = hydraulic radius; V = mean velocity; and e = specific energy. In Col.[a] 9, $\frac{1}{(n\,v)^2} = \frac{1}{2.2082\,R^{4/3}}$. In Cols. 10 and 11, S = friction slope and S_o = bed slope; and in Cols. 12 and 13, Δx = a finite length of channel, and $x = \Sigma\,\Delta x$, the distance from the starting point of the surface curve.

y (1)	A (2)	p (3)	R (4)	V (5)	$\frac{V^2}{2g}$ (6)	e (7)	Δe (8)	$\frac{1}{(n\,v)^2}$ (9)	S (10)	$S_o - S$ (11)	Δx (12)	x (13)
4.0	20.0	13.0	1.54	2.02	0.063	4.063	....	....	....	....	....	0
3.5	17.5	12.0	1.46	2.30	0.082	3.582	0.481	0.273	0.00052	0.00248	194	194
3.0	15.0	11.0	1.36	2.69	0.112	3.112	0.470	0.301	0.00079	0.00221	212	406
2.5	12.5	10.0	1.25	3.23	0.162	2.662	0.450	0.336	0.00126	0.00174	258	664
2.0	10.0	9.0	1.11	4.04	0.254	2.254	0.408	0.394	0.00230	0.00070	583	1,247
1.85	9.25	8.70	1.06	4.37	0.297	2.147	0.107	0.419	0.00288	0.00012	890	2,137

[a] "Handbook of Hydraulics," by Horace W. King, McGraw-Hill Book Co., Inc., New York, N. Y., 3d Ed., 1939, p. 309.

in the direction of increasing depths. There is a gain in specific energy downstream as shown in Fig. 5 due to the accelerating force of gravity. Fig. 6 shows the surface curve taken from Fig. 3. The circles are points given in Table 4. The agreement is practically perfect.

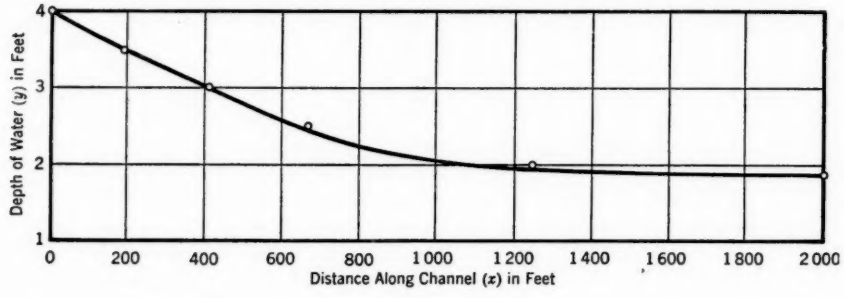

FIG. 6

Example 1 is the same as that presented by Mr. Bakhmeteff[2] in 1932, who recognized that this surface curve includes a hydraulic jump but did not define it.

For this example the bed slope is less than the friction slope, the depths are sub-critical and sub-normal, and specific energy diminishes downstream, resulting in the formula,

$$\Delta x = \frac{\Delta e}{S - S_o} \qquad (22)$$

Fig. 7 shows the actual surface curve. The computations began at the critical depth 4.49 and extended upstream using depth decrements of 0.10 ft on account of the rapidly changing curvature. The circles are points given by Mr. Bakhmeteff.

To locate the position of the hydraulic jump it was necessary to compute the surface curve downstream from the critical depth where super-critical depths obtain. At points upstream from the critical depth the values of $P + \frac{Q}{g} V$ were computed from both the sub-critical and super-critical curves, where P is the hydrostatic pressure in the channel cross section and $\frac{Q}{g} V$ is the momentum of the water, as set forth by Julian Hinds,[11] M. ASCE.

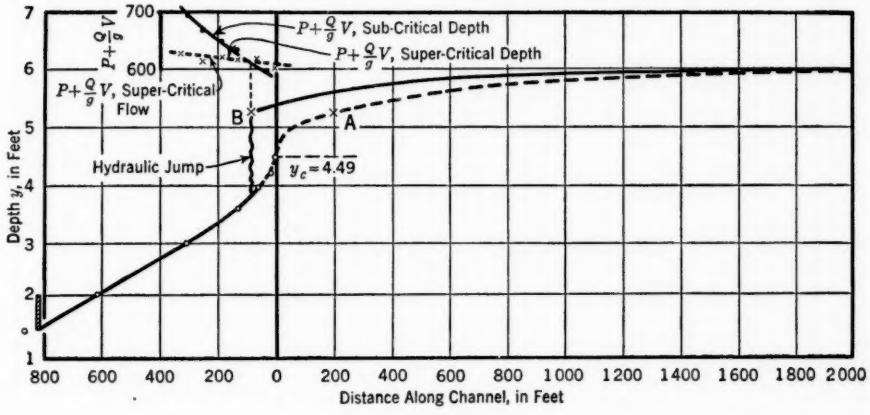

FIG. 7.

The height of the jump was determined by the method given by the writer[12] to be 4.24 ft above channel bed. The super-critical surface curve was then moved bodily upstream from point A to point B. The solid lines in Fig. 7 therefore represent the actual surface curve for the author's Example 1 neglecting energy losses.

The computation of surface curves from a varied-flow function, as used by the author, Messrs. Bresse, Bakhmeteff, and others, practically precludes the possibility of taking all the factors into account. On the other hand, they are readily applied in the step-by-step method. For example, if the writer were trying to find the surface curve for Example 1 he would compute it from an equation something like

$$\Delta x = \frac{\Delta(y + 1.05\, h_V) - 0.20\, \Delta h_V}{S - S_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

in which h_V equals the velocity head of the mean velocity.

[11] "The Hydraulic Jump and Critical Depths in the Design of Hydraulic Structures," by Julian Hinds, *Engineering News-Record*, Vol. 85, 1920, p. 1034.

[12] "The Hydraulic Jump in Standard Conduits, by J. C. Stevens, *Civil Engineering*, October, 1933, p. 565.

A much closer approximation to the true surface curve of Example 1 from the sluice gate to the foot of the hydraulic jump would thus be obtained, that is, by adding 5% to the kinetic energy of the mean velocity and then assuming only 80% velocity head recovery in expanding areas.

MAURICE BISHAY[13] AND H. E. BABBITT,[14] M. ASCE.—An illustrative problem involving the computation of a backwater curve has been solved by the author's method and five other methods. Results of this computation are shown in Table 5 and in Fig. 8. Comments are given on the relative accuracy of the various methods and on the relative tediousness of the work involved in each method.

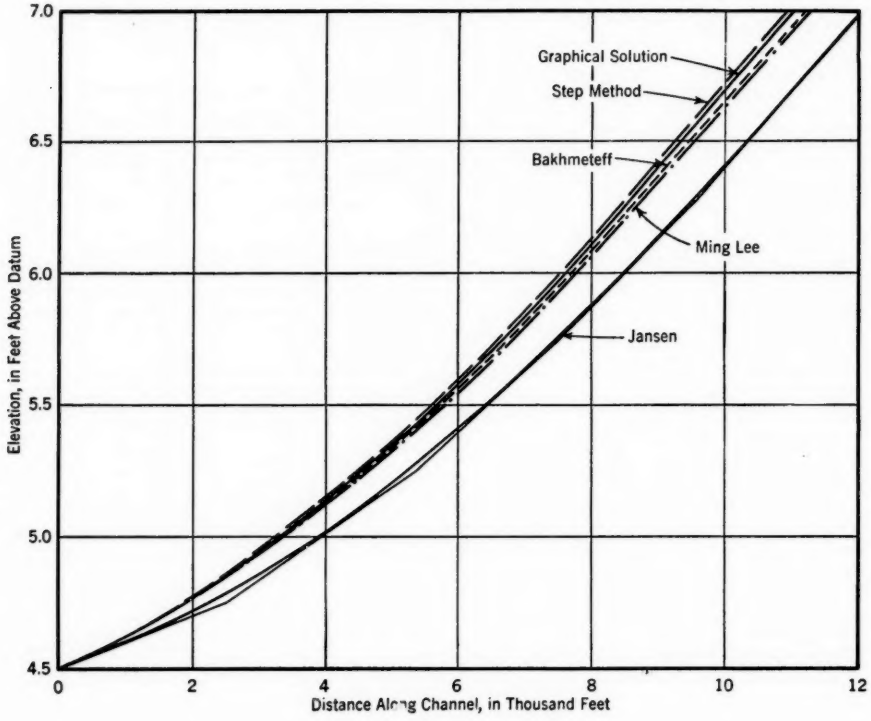

FIG. 8

Problem.—The problem selected for solution requires the construction of the backwater curve for a stream flowing under a sluice gate into a rectangular channel whose bottom width is 10 ft. The slope of the bottom of the channel is 0.0004 and the value of Manning's n is 0.01486. The sluice gate is regulated so that the rate of discharge is 136 cu ft per sec, with a depth at the vena contracta of 7.0 ft.

[13] Graduate Student, Univ. of Illinois, Urbana, Ill.
[14] Prof. of San Eng., Univ. of Illinois, Urbana, Ill.

Preliminary Steps in Solution.—The depth at uniform flow, computed by the Manning formula, is found to be 4.0 ft. Critical depth is found to be 1.79 ft. Hence, the surface curve will be of the M-1 type of gradually varied flow in a rectangular channel.

Solution by Graphical Integration.—As explained by Hunter Rouse,[15] M. ASCE, but using the American Standards Association hydraulic symbols (ASA—Z10.2—1942):

$$\Delta x = \frac{\Delta E_s}{S_o - S_f} \ldots (24)$$

or

$$\int_a^b dx = \int_a^b \frac{dE_s}{S_o - S_f} \ldots (25)$$

TABLE 5.—COMPUTED DISTANCES (IN FEET) ALONG THE CHANNEL, BELOW A DEPTH OF 4.5 FT COMPARED BY SIX METHODS

Depth (ft)	Graphical (1)	Step (2)	Bakhmeteff (3)	Lee (4)	Mononobe (5)	Jansen (6)
5.0	3,281	3,250	3,336	3,344	....	....
5.5	5,683	5,580	5,723	5,737	....	....
6.0	7,638	7,542	7,708	7,724	....	....
6.5	9,386	9,299	9,469	9,483	....	....
7.0	11,025	10,933	11,204	11,230	10,170	12,088
Deviation[a]	0	−0.83	+1.62	+1.8	−7.75	+9.6

[a] Percentage deviation from the graphcial method at a depth of 7.0 ft.

in which a and b represent any two points on the backwater curve. That is, the distance between two points on the backwater curve can be obtained by the integration of $\frac{dE_s}{S_o - S_f}$ between points a and b. The value of S_f can be obtained from the expression,

$$V = C\, R^{2/3}\, S^{1/2}{}_f \ldots (26)$$

TABLE 6.—COMPUTATION OF THE BACKWATER CURVE BY GRAPHICAL INTEGRATION

Depth (ft)	Hydraulic radius (ft) (1)	Specific energy[a] (2)	Slope ($S_f \times 10^{-4}$) (3)	$\frac{1}{S_o - S_f} \times 10^{+3}$ (4)	Area under curve (sq ft) (5)	Distance along channel[b] (6)
4.5	2.37	4,643	2.92	9.26	0	0
5.0	2.50	5,115	2.18	5.50	3,281	3,281
5.5	2.62	5,595	1.70	4.35	2,402	5,683
6.0	2.72	6,080	1.35	3.775	1,955	7,638
6.5	2.82	6,568	1.10	3.45	1,748	9,386
7.0	2.92	7,059	0.902	3.23	1,639	11,025

[a] In Col. 2, specific energy is $y + \frac{V^2}{2g}$. [b] Distance, in feet, along the channel below a depth of 4.5 ft.

If the curve of E_s is plotted against $\frac{1}{S_o - S_f}$ to any convenient scale, the distance between two points on the backwater curve can be computed by measuring the area under the $\left(E_s\text{-to-}\frac{1}{S_o - S_f}\right)$-curve. Such a curve has been drawn and the results of the computation by the graphical method are shown in Table 6.

Solution by Step Method.—The expression of the distance between two points on a backwater curve can be derived from the Bernouilli theorem and expressed

[15] "Elementary Mechanics of Fluids," by Hunter Rouse, John Wiley & Sons, Inc., New York, N. Y., 1946, pp. 80–82.

in standard nomenclature as

$$\Delta x = \frac{\Delta \epsilon_s}{S_o - \frac{S_{f1} - S_{f2}}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Results of the computation of the backwater curve by this expression applied to the illustrative problem are shown in Table 7.

TABLE 7.—COMPUTATION OF THE BACKWATER CURVE BY THE STEP METHOD

Depth (ft)	Specific energy ϵ_s (ft)	$\Delta\epsilon_s$ (ft)	Friction slope[a]	Mean $S_f \times 10^{-4}$	$\frac{1}{S_o - S_f} \times 10^{-3}$	Δx (ft)	Distance along channel[b]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
4.5	4,643		2.92				0
5.0	5,115	0.472	2.18	2.55	6.9	3,250	3,250
5.5	5,595	0.480	1.70	1.94	4.86	2,330	5,580
6.0	6,080	0.485	1.35	1.525	4.05	1,962	7,542
6.5	6,568	0.488	1.10	1.225	3.60	1,757	9,299
7.0	7,059	0.491	0.902	1.001	3.33	1,634	10,933

[a] Friction slope in Col. 3 is $S_f \times 10^{-4}$. [b] Distance in feet along the channel below a depth of 4.5 ft.

Solution by Direct Integration, Using the Bakhmeteff Method[16].—Mr. Bakhmeteff has derived the expression,[3]

$$x_b - x_a = \frac{y_o}{S_o}\{\gamma_b - \gamma_a - (1 - \beta)[\beta(N, \gamma_b) - \beta(N, \gamma_a)]\} \ldots (28)$$

which can be solved by the aid of tables for N and for γ computed by Mr. Bakhmeteff, proceeding by small increments of depth so that β may be assumed to be constant within each interval. Results of computations by Mr. Bakhmeteff's method are compiled in Tables 8 and 9.

The Mononobe Method[17].—In this method the assumption is made that both the area A and the wetted perimeter P_w may be expressed as monomial functions of the depth y. For example, P_w is equal to a constant times y^l and A is equal to a constant times $y^{M/2}$. The expression of N. Mononobe, based on the preceding assumptions, is

$$x_2 - x_1 = \frac{y_o}{S_o}\left\{\phi_1(r_2) - \phi(r_1) - \frac{V^2_o M}{2 g y_o}[\phi_2(r_2) - \phi_2(r_1)]\right\} \ldots (29)$$

TABLE 8.—HYDRAULIC PROPERTIES OF A RECTANGULAR CHANNEL

Depth (ft)	Area (sq ft)	Wetted perimeter (ft)	Hydraulic radius (ft)	$R^{2/3}$	Conveyance K	Constant β
	(1)	(2)	(3)	(4)	(5)	(6)
4	40	18	2.22	1.71	6,840	0.0912
5	50	20	2.50	1.85	9,250	0.0854
6	60	22	2.72	1.95	11,760	0.0796
7	70	24	2.92	2.05	14,350	0.0746

in which

$$r = \frac{y}{y_o}; \phi_1(r) = \int_{1.001}^{r} \frac{r^{C_o} dr}{r^{C_o} - 1}; \phi_2(r) = \int_{1.001}^{r} \frac{r^{C_o - M - 1} dr}{r^{C_o} - 1} \ldots\ldots (30)$$

and

$$C_o = \frac{5}{3} M - \frac{4}{3} l \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

[16] "Hydraulics of Open Channels," by B. A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, Chapter VIII.

[3] "An Investigation of the Backwater Profile for Steady Flow in Prismatic Channels," by W. M. Lansford and W. D. Mitchell, *Bulletin Series No. 381*, Univ. of Illinois, Urbana, Ill., 1949.

[17] "Backwater and Drop-Down Curves for Uniform Channels," by Nagaho Mononobe, *Transactions*, ASCE, Vol. 103, 1938, p. 950.

The work involved in the computation of the complete backwater curve for the illustrative example is so tedious that only the distance between the depths of 4.5 ft and 7.0 ft has been located. It was found by means of the equation

TABLE 9.—COMPUTATION OF THE BACKWATER CURVE BY THE BAKHMETEFF METHOD

Depth (ft)	$\gamma = \frac{y}{y_o}$ (1)	$\gamma_b - \gamma_a$ (2)	$1 - \beta$ (3)	$1 - \beta$ (average) (4)	$\beta(N, \gamma)$ (5)	$\Delta\beta(N, \gamma)$ (6)	Δx (7)	Distance[a] (ft) (8)
4.5	1.125	0.125	0.9121		0.8045			0
5.0	1.250	0.125	0.9146	0.9133	0.5760	−0.2285	3,336	3,336
5.5	1.375	0.125	0.9174	0.9160	0.4520	−0.1240	2,387	5,723
6.0	1.500	0.125	0.9204	0.9190	0.3720	−0.0800	1,985	7,708
6.5	1.625	0.125	0.9229	0.9216	0.3165	−0.0555	1,761	9,469
7.0	1.750	0.125	0.9254	0.9242	0.264	−0.0525	1,735	11,204

[a] Distances in feet, along the channel, below a depth of $y = 4.5$ ft.

and charts that

$$x_2 - x_1 \frac{4.0}{0.004}\left[3.00 - 1.93 - \frac{(3.4)^2\,(2)}{64.4\,(4)}\,(2.42 - 1.83)\right] = 10{,}170 \text{ ft}$$

Method of Ming Lee[18].—Mr. Lee's basic expression is in the form,

$$\Delta x = \frac{y_o}{S_o}\left[\Delta r - \Delta\beta\,(N, \gamma) + \frac{V^2{}_o}{2\,g\,y_o}\Delta\beta\left(\frac{N}{M}, \frac{A^2{}_o}{A^2}\right)\right] \quad \ldots\ldots\ldots (32)$$

in which all the nomenclature is "standard," except values of $\beta\left(\frac{N}{M}, \frac{A^2{}_o}{A^2}\right)$

TABLE 10.—COMPUTATION OF BACKWATER CURVE BY THE METHOD OF MING LEE

Depth (ft)	$\Delta\gamma$ (1)	$\Delta\beta(N, \gamma)$ (2)	$\frac{A^2{}_o}{A^2}$ (3)	$\beta\left(\frac{N}{M}, \frac{A^2{}_o}{A^2}\right)$ (4)	$\Delta\beta\left(\frac{N}{M}, \frac{A^2{}_o}{A^2}\right)$ (5)	Δx (6)	Distance[a] (ft) (7)
4.5			0.790	1.308			0
5.0	0.125	−0.2285	0.640	0.884	−0.424	3,344	3,344
5.5	0.125	−0.1240	0.530	0.667	−0.217	2,392	5,737
6.0	0.125	−0.0800	0.445	0.527	−0.140	1,987	7.724
6.5	0.125	−0.0555	0.378	0.427	−0.100	1,760	9,483
7.0	0.125	−0.0525	0.330	0.363	−0.064	1,746	11,230

[a] Distances in feet along the channel below a depth of $y = 4.5$ ft.

which are given by Mr. Lee in a table. Using Mr. Bakhmeteff's and Mr. Lee's tables, values of the coordinates of points on the curve have been computed, with the results shown in Table 10.

[18] "Steady, Gradually Varied Flow in Uniform Channels on Mild Slopes" by Ming Lee, thesis presented to the University of Illinois, Urbana, Ill., in 1947, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

The Jansen Method.—Results of computations of the slopes of the curve at various points by Mr. Jansen's method are shown in Table 11. In sketching curves by this method the tangents are made to intersect at the midpoint between contiguous depths. The backwater curve resulting is shown in Fig. 8.

Summary.—Theoretically, the graphical method yields the most accurate results because no assumptions are made concerning the conditions in the integration procedure, other than those made in the derivation of the equation of gradually varied flow. However, the work involved in this method is tedious. The work involved in the step method is relatively simple and the results are of sufficient accuracy for most purposes. The methods of direct integration proposed by Mr. Bakhmeteff and by Mr. Lee give similar results and are less tedious than the graphical method, but may be less accurate because of assumptions made in the computation of the integral functions. In the Bakhmeteff method, the empirical relationship between the conveyance and the depth holds for a wide range of depths. However, there may be appreciable changes in the velocity heads that are significant. Hence, the limitations imposed by assuming β to be constant make it necessary to restrict the computations to small increments of depth. In Mr. Lee's method β is regarded as a function of the depth, for practical purposes, so that the integration limits may be taken over wider ranges of depth. Under such conditions the Lee method may be convenient to use.

TABLE 11.—COMPUTATION OF BACKWATER CURVE BY THE JANSEN METHOD

Depth, y (ft)	Factor, $\mu \times 10^{-5}$ (1)	Factor, $i \times 10^{-15}$ (2)	Slope (3)
4.5	15.60	11.0	0.000101
5.0	11.82	8.0	0.000177
5.5	9.17	6.03	0.000288
6.0	7.29	4.635	0.000260
6.5	5.91	3.64	0.000280
7.0	4.92	1.92	0.000306

In the Mononobe method the computations are more direct in that the distance along the channel between any two sections may be determined in one step. The integration of the gradually varied flow equation is simplified by making the assumptions that both the wetted perimeter and the area can be expressed as monomial functions of the depth. The drawback to the Mononobe method is the difficulty of using the graphical charts prepared by him, and the inherent errors resulting from interpolations in the reading of the chart.

Mr. Jansen's method and Mr. Mononobe's method differ from each other and the other four methods investigated, in the results obtained in the illustrative problem. The difficulties with these two methods lie mainly in the reading of the intersection of lines and curves. In the Jansen method, in order to obtain a uniform curve the computed tangents are made to intersect at the midpoint between contiguous depths. The accuracy with which the location of such points is determined fixes the degree of approximation in the shape of the resulting curve.

K. C. WU,[19] A. M. ASCE.—The determination of water surface profiles in open channels carrying steady nonuniform flow was first solved by J. A. C.

[19] Lecturer in Civ. Eng., Univ. of Hong Kong, Hong Kong.

Bresse[20,21] for an infinitely wide rectangular channel as early as 1860. In 1912 Mr. Bakhmeteff first suggested a generalized solution for channels of any shape which, in 1932, he published[2] giving many numerical solutions. In 1948 Phillip Z. Kirpich,[22] A. M. ASCE, proposed a dimensionless formula with a set of curves for the solution of the hydraulic exponent, n_B, for rectangular and trapezoidal channels. In 1949 M. E. Von Seggern,[23] A. M. ASCE, introduced another exponent m, in order to facilitate the integration of the equation of steady nonuniform flow. Mr. Jansen's paper is a further extension, and, at the same time, a new and ingenious approach to the same problem. As stated by the author, the method is applicable only to rectangular and trapezoidal channels of symmetrical shapes.

Eqs. 10, 11, and 12 can be extended so that values of the u-functions and j-functions are in dimensionless forms. Substituting $\alpha = y/b$ and simplifying, Eq. 10 becomes

$$u = \frac{(1 + P\,\alpha)^{4/3}}{b^{16/3}\,[\alpha\,(1 + \phi\,\alpha)]^{10/3}} \qquad (33a)$$

and Eq. 11 becomes

$$j = \frac{1 + 2\,\phi\,\alpha}{b^5\,[\alpha\,(1 + \phi\,\alpha)]^3} \qquad (33b)$$

Next, assume that $u\,b^{16/3} = U$ and $j\,b^5 = J$ (both of which are dimensionless functions depending only on α and ϕ since P is also a function of ϕ). Eqs. 33 then become

$$U = \frac{(1 + P\,\alpha)^{4/3}}{[\alpha\,(1 + \phi\,\alpha)]^{10/3}} \qquad (34a)$$

and

$$J = \frac{1 + 2\,\phi\,\alpha}{[\alpha\,(1 + \phi\,\alpha)]^3} \qquad (34b)$$

Eq. 12 can then be written as

$$\frac{dy}{dx} = \frac{S_o - \frac{N}{b^{16/3}}\,U}{1 - \frac{G}{b^5}\,J} = \frac{S_o - N'\,U}{1 - G'\,J} \qquad (35)$$

in which

$$N' = \frac{N}{b^{16/3}} = \frac{n^2\,Q^2}{2.208\,b^{16/3}} \qquad (36a)$$

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[20] "Mechanique Appliquée," by J. A. C. Bresse, Mallet-Bachelier, Paris, France, 1860.

[21] "Hydraulics of Steady Flow in Open Channels," by S. M. Woodward and C. J. Posey, John Wiley & Sons, Inc., New York, N. Y., 1941.

[22] "Dimensionless Constants for Hydraulic Elements of Open Channel Cross-Sections," by Phillip Z. Kirpich, *Civil Engineering*, October, 1948, p. 47.

[23] "Integrating the Equations of Nonuniform Flow," by M. E. Von Seggern, *Transactions*, ASCE, Vol. 115, 1950, p. 71.

and

$$G' = \frac{G}{b^5} = \frac{Q^2}{g\,b^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36b)$$

All terms on the extreme right side of Eq. 35 are dimensionless. For any prismatic trapezoidal channel (including rectangular channels), both N' and G'

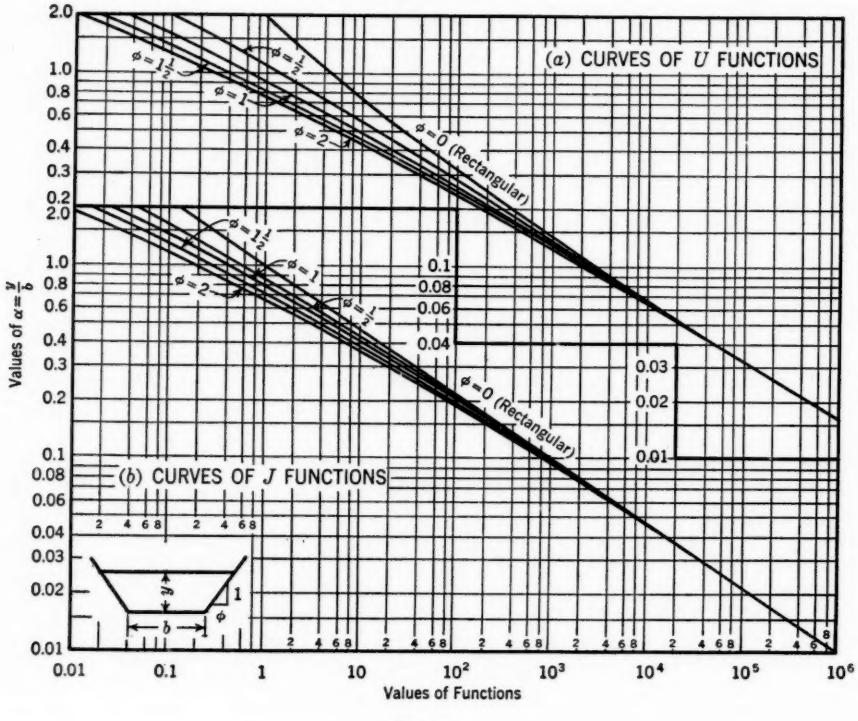

FIG. 9

can be easily calculated, and U and J can be obtained either from curves (such as Fig. 9) or from prepared tables; and the slope dy/dx between the desired sections can be computed by Eqs. 35 and 36.

J. PIETRKOWSKI[24].—An equation for the slope of the water surface in a trapezoidal channel for steady nonuniform flow has been developed by the author. The practical solution of the equation requires the use of special tables. The surface curve itself must be found from various values of the slope either graphically or numerically. In order to test the advantage of the proposed method, it seems logical to compare it with the usual tabulation method.

[24] Director of Water Section, Public Works Dept., Tel-Aviv, State of Israel.

The basic relation (Fig. 10) is expressed by

$$dx = \frac{h_1 - h_2}{S - S_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

With h_1 and S_o known, textbooks usually assume a definite value for dx to find values h_2 and S that satisfy Eq. 37. The method thus becomes subject to trial-and-error, which can be avoided by solving Eq. 37 for dx. An inspection of the right side of Eq. 37 shows that h_1 is given, if shooting flow exists. To determine h_2 assume a reasonable value, $y_2 = y_1 + \Delta y$. Then $A_2 = f(y_2)$, and $V_2 = Q/A_2$. The slope of the channel bottom, S_o, is given and the energy gradient can be computed by the relation

$$S = \frac{V^2_m\, n^2}{(R_m)^{4/3}} \quad \ldots\ldots\ldots\ldots (38)$$

assuming that

$$V_m = \sqrt{\frac{V^2_1 + V^2_2}{2}} = \frac{1}{n}\,(R_m)^{2/3}\, S^{1/2} \ldots (39)$$

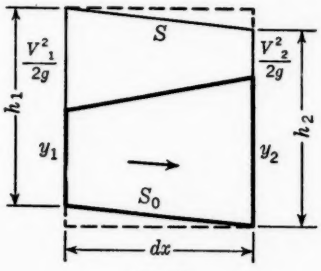

FIG. 10

The application of the method is shown in Table 12, using Example 1.

The values are converted approximately into metric system, that is, b = 6.0 m; y_1 = 0.45 m; Q = 30 cu m per sec; and, $n^2 \times 10^4$ = 1.69. The calculation was made by slide rule.

The result checks sufficiently with Fig. 2. The tabulation method can be used for any uniform cross section and gives a direct solution for the water surface whereas the proposal by the author is restricted to a special shape of cross section, requires previous tabulations, and includes (for the graphical solution) flat and, therefore, inaccurate intersections of lines. The tabulation method, therefore, seems to be simpler and most easily visualized.

TABLE 12.—DETERMINATION OF SURFACE SLOPE, EXAMPLE 1 (METRIC UNITS[a])

y_1 (1)	V_1 (2)	h_1 (3)	y_2 (4)	V_2 (5)	h_2 (6)	h_1—h_2 (7)	V^2_m (Eq. 39) (8)	R_m (9)	$(R_m)^{4/3}$ (10)	S (Eq. 38) (11)	S—S_o (12)	x (Eq. 37) (13)
0.45	11.1	6.75	0.60	8.35	4.15	2.60	96.3	0.44	0.33	0.0495	0.0485	53.5
0.60	8.35	4.15	0.75	6.65	3.00	1.15	57	0.54	0.44	0.0220	0.0210	55.0
0.75	6.65	3.00	0.90	5.55	2.47	0.53	37.5	0.64	0.55	0.0115	0.0105	50.5

[a] 1 meter (Cols. 1, 3, 4, 6, 7, 9, and 13) = 39.37 in.; 1 meter per second (Cols. 2 and 5) = 196.85 ft per min = 2.23693 miles per hr; and 1 meter raised to the four-thirds power (Col. 10) = 133.8 $in.^{4/3}$.

In the case of the more complicated problems actually encountered in practice, the idea of finding the surface line by calculating the slope first, can be used with advantage in a tabulation method. For example, the writer has been required to calculate the backwater profile for a trapezoidal river channel with

gradually increasing width and dikes for a double profile. The water-filled cross section in that case is not a function of the depth alone. It is also necessary to know the place along the channel at which this depth occurs. The distance between control sections is assumed and, for each of these sections, the elevation and slope of the energy grade line is determined, which will result in a continuous line without steps at the intersections.

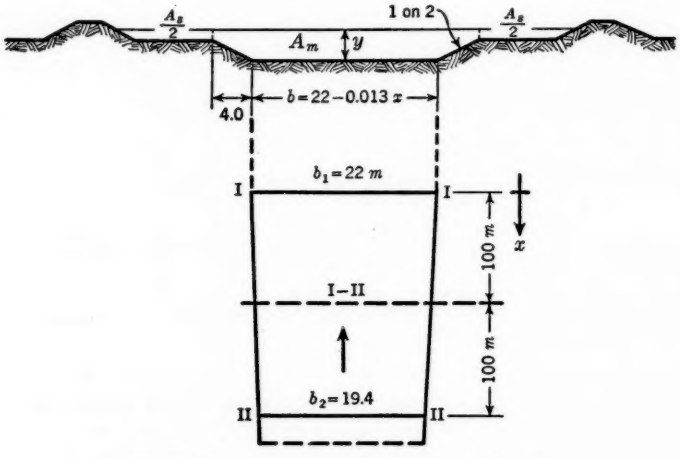

FIG. 11.—DOUBLE PROFILE WITH CHANGING WIDTH

As an illustration, assume a cross section such as Fig. 11 in which S_o = 1:1250; Q = 90 cu m per sec (263 acre-ft per hr); n = 0.025; the flow conditions at section 1 are assumed known, with y_1 = 200 m; and S_1 = 1:1,000. The discharge conditions at control sections are examined over a distance of Δx = 200 m, with the energy grade lines interescting at Δx 2; thus:

Symbol	Computation	Metric units	English units
A_1	2.0 (22.0 + 4.0)	52 square meters	560 square feet
$\frac{1}{n}$	$\frac{1}{0.025}$	40	40
$\frac{1}{n^2}$	...	1600	1600
R_1	...	1.68 meters	66 inches
$(R_1)^{2/3}$	...	1.41 meters$^{2/3}$	16.4 inches$^{2/3}$
S_1	...	1/1000	1/1000
$\sqrt{S_1}$	...	1/31.6	1/31.6
V_1	$\frac{40 \times 1.41}{31.6}$	1.78 meters per second	351 feet per minute
Q	1.78 × 52	93 cubic meters per second	272 acre-feet per hour

Symbol	Computation	Metric units	English units
$\frac{V^2_1}{2g}$	...	0.16 meter	6.3 inches
y_1	...	2.00 meters	78.7 inches
$H_1 - h_1$	0.16 + 2.00	2.16 meters	85.0 inches
$S_1 \frac{dx}{2}$	$\frac{100}{1000}$	0.10 meter	3.9 inches
H_{1-2}	2.16 + 0.10	2.26 meters	88.9 inches
$S_o \frac{dx}{2}$	$\frac{100}{1250}$	0.08 meter	3.1 inches
h_{1-2}	2.26 − 0.08	2.18 meters	85.8 inches

It is assumed that $y_2 = 2.03$ m. In this case, the correct value is found most easily by trial and error; but it is possible to calculate, beforehand, a table or diagram of several values from which that one is selected which will

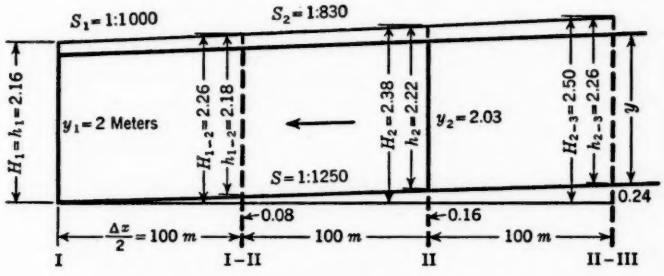

FIG. 12.—DETERMINATION OF WATER SURFACE FLOW

fulfill the required conditions. The influence of the side channels is neglected, thus (see Fig. 12):

Symbols	Computations	Metric units	English units
A_2	2.03 (19.4 + 4.0) 0.03 × 4.0	47.6 square meters	512 square feet
R_2	...	1.67 meters	65.7 inches
$(R_2)^{2/3}$	...	1.41 meters$^{2/3}$	16.4 inches$^{2/2}$
$(R_2)^{4/3}$	...	1.99 meters	78.4 inches
V_2	$\frac{93}{47.6}$	1.95 meters per second	384 feet per minute
V^2_2	...	3.82	...
$\frac{V^2_2}{2g}$	...	0.19 meter	7.5 inches
y_2	...	2.03	80.0 inches
h_2	0.19 + 2.03	2.22 meters	87.5 inches
$\frac{1}{S_2}$	$\frac{(R_2)^{4/3}}{V^2_2 n^2}$	830	...

Symbols	Computations	Metric units	English units
$S_o\,\Delta x$	$\frac{200}{1250}$	0.16 meter	6.3 inches
H_2	2.22 + 0.16	2.38 meters	93.8 inches
$-\,S_2\frac{\Delta x}{2}$	$\frac{100}{830}$	0.12 meter	4.7 inches
H_{1-2}	2.38 − 0.12	2.26 meters	89.1 inches
H_{2-3}	2.38 + 0.12	2.50 meters	98.4 inches
h_{2-3}	2.50 − 0.24	2.26 meters	89.1 inches

The value of $H_{1-2} = 2.26$ meters (89 in.) agrees with that found in the preceding computation.

With the continuation of the calculation to sections upstream it will be necessary to take the flow in the side channels into account. It will be sufficient to distribute the discharge between the main channel and the side channels in accordance with the relation,

$$\frac{Q_m}{Q_s} = \frac{A_m \times (R_m)^{2/3} \times n_S}{A_S \times (R_S)^{2/3} \times n_m} \qquad (40)$$

and then to continue the calculation for the main channel only.

Separation losses and eddy losses can be accounted for by a suitable selection of the friction value n for each stretch dx, separately.

To summarize: The basic idea offered in this paper should not be wasted on simple problems for which the existing methods are good enough, but should be used for more complicated cases, where the existing trial-and-error methods are inadequate.

ROBERT B. JANSEN,[25] J.M. ASCE.—A lucid verification of the results yielded by the suggested graphical procedure has been presented by Mr. Stevens. It should not be surprising that curves plotted by this method conform with those computed by the equation of Mr. Husted, since both methods of solution are based on the classic law of the conservation of energy, and both make use of weighted energy gradients to define the water slope in a finite increment of channel. Any discrepany observed between the two curves should be primarily a reflection of differences in assumptions of energy loss and of channel increment.

If Eq. 2 be considered in finite terms, it can be written as

$$\frac{\Delta h}{\Delta x} = \frac{\Delta z}{\Delta x} + \frac{\Delta y}{\Delta x} + \frac{\Delta}{\Delta x}\left(\frac{V^2}{2g}\right) \qquad (41)$$

Multiplying Eq. 41 by Δx, and substituting $S\,\Delta x = \Delta h$ and $S_o\,\Delta x = \Delta z$, it takes the form:

$$S\,\Delta x = S_o\,\Delta x + \Delta y + \Delta\left(\frac{V^2}{2g}\right) \qquad (42)$$

[25] Civ. Engr. Associate, Bureau of Eng., City of Los Angeles, Calif.

Eq. 42 can be transformed to

$$\Delta x = \frac{\Delta y + \Delta\left(\frac{V^2}{2g}\right)}{S - S_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

which is the same as Eqs. 20 and 22.

In deriving Eq. 43, friction was the only cause of energy loss included. In order to take eddy loss also into account, the term $K\Delta\left(\frac{V^2}{2g}\right)$ should be subtracted from the right side of Eq. 42 so that the final equation becomes

$$\Delta x = \frac{\Delta y + \Delta\left(\frac{V^2}{2g}\right)(1 - K)}{S - S_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

It is lamentable that relationships for energy loss in an open channel have not been developed for nonuniform flow. The use of the Chezy equation in this case is pardonable only because no satisfactory substitute is known. Although it is generally accepted that the loss in diverging flow is greater than that in uniform flow, no reliable mathematical statement of the difference is available in the technical literature. The application of the eddy loss coefficient, K, is a rough attempt to compensate for the lack of a proper formula. Mr. Stevens' selection, $K = 0.20$, is a value frequently applied in practice.

To inject an eddy-loss factor such as this into the writer's equations requires only a simple modification. If the velocity head differential in Eq. 2 is multiplied by the quantity $(1 - K)$ as suggested, it can be shown that Eq. 5 will become

$$\frac{dy}{dx} = \frac{S_o - \frac{Q^2}{A^2 C^2 R}}{1 - (1 - K)\frac{Q^2 T}{A^3 g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

and Eq. 12 will be changed to

$$\frac{dy}{dx} = \frac{S_o - N u}{1 - (1 - K) G j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

Since both the coefficient K and the factor α, as applied by Mr. Stevens, are dimensionless, their introduction into the writer's equations calls for a very minimum of effort. This is contradictory to Mr. Stevens' opinion that "The computation of surface curves from a varied-flow function as used by the author * * * practically precludes the possibility of taking all the factors into account."

The writer does not consider it necessary here to defend the merits of the Manning formula, particularly since, in this case, it yields results close to those given by the Kutter relation. However, Mr. Stevens may find some argument from other quarters on his assertion that the Manning expression is dimensionally incorrect since the dimension of the roughness factor n is a topic for debate.

Contrary to the opinion of Mr. Stevens and Mr. Pietrkowski, the tables of the depth functions, u and j, are not essential to the method presented in the paper—no more so, in fact, than they would be to the Husted procedure or to methods advocated by others. It will be noted that these functions are simply

$$u = \frac{p^{4/3}}{A^{10/3}} = \frac{1}{A^2 R^{4/3}} \quad \ldots\ldots (47)$$

and

$$j = \frac{T}{A^3} \quad \ldots\ldots (48)$$

The Manning value of the energy gradient is obtained by multiplying u by the term N, thus,

$$S = N u = \left(\frac{n^2 Q^2}{2.208}\right)\left(\frac{1}{A^2 R^{4/3}}\right) \quad \ldots\ldots (49)$$

The significance of the function j in defining the change in velocity head has already been indicated in Eq. 3. That relation can be rewritten to give

$$d\left(\frac{V^2}{2g}\right) = - G j \, dy \quad \ldots\ldots (50)$$

It is apparent from Eqs. 49 and 50 that the application of the depth functions u and j to the computation of surface curves by step procedures, such as those recommended by Mr. Stevens, would be more than a minor convenience.

In order to illustrate the general applicability of these functions, reference is made to a graphical method described by S. M. Woodward, Hon. M. ASCE, and C. J. Posey,[21] M. ASCE. Their general equation can be written

$$dx = f(y) \, dy \quad \ldots\ldots (51a)$$

in which

$$f(y) = \frac{1 - G j}{S_o - N u} \quad \ldots\ldots (51b)$$

The relation between Eqs. 51 and Eq. 12 is obvious.

In the graphical solution outlined by Messrs. Woodward and Posey, values of $f(y)$ are obtained for the range of depths covered by the water surface profile. These values of $f(y)$ are then plotted as ordinates, versus depths as abscissas. The area under the resulting curve between y_1 and y_2 is the distance between the corresponding stations.

It should be emphasized that the functions u and j are of general purpose, and can be applied to advantage not only in the writer's solution but in the Woodward-Posey method and the Husted method as well. This point can best be clarified by considering a typical problem.

Example 3.—Water is flowing from a reservoir into a trapezoidal spillway channel whose bottom width is 15 ft. The slope of the spillway floor is $S_o = 0.0530$ and the roughness of the lining is given by $n = 0.025$. The rate of flow is 1,000 cu ft per sec. The side slopes of the channel are both 1 on 1.5.

The depth of the control section is given by $y_c = 4.43$ ft. The constant terms in the equations are: $N = \frac{n^2 Q^2}{2.208} = \frac{(0.025)^2 (1{,}000)^2}{2.208} = 283$ and $G = \frac{Q^2}{g} = \frac{(1{,}000)^2}{32.2} = 31{,}100$.

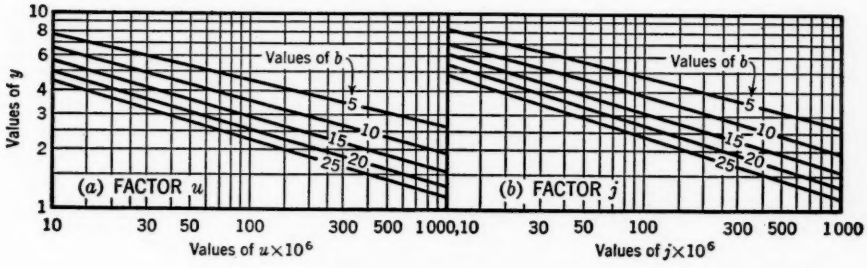

FIG. 13.—FACTORS FOR SIDE SLOPES OF 1 ON 1.5

The values of the functions u and j are obtained from Fig. 13 which was derived from the writer's tables[2a] of the depth factors for side slopes of 1 on 1.5. When $y = 4.0$ ft, $\frac{dy}{dx} = \frac{S_o - N\,u}{1 - G\,j} = \frac{0.0530 - 283\,(35.0 \times 10^{-6})}{1 - 31{,}100\,(45.6 \times 10^{-6})} = -0.1032$.

The surface slopes at other depths are obtained similarly. A summary of the calculations required for direct plotting is presented in Table 13(*a*). The curve of the water surface in the spillway is drawn in Fig. 14.

TABLE 13.—COMPUTATIONS FOR EXAMPLE 3

Depths, y or y_1	(*a*) DETERMINATION OF SURFACE SLOPES[a]				(*b*) SOLUTION BY THE HUSTED METHOD			
	Factor[b] u	Factor[b] j	Slope dy/dx	$f(y)$	Depth y_2	Average Factors[b]		Δx
						u	j	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
4.43	...	32.1	∞	0	4.00	29.6	38.9	2.0
4.00	35.0	45.6	−0.1032	9.68	3.50	45.9	58.6	10.3
3.50	56.7	71.6	−0.0301	33.2	3.00	77.5	95.8	31.8
3.00	98.2	119.9	−0.0092	108.6	2.75	115.9	139.9	41.5
2.75	133.6	159.9	−0.0038	261	...	...	...	...
2.50	186.6	218.5	0.0000	∞	Summation, in feet.....			85.6

[a] Values of $f(y)$ in Col. 5 are by C. J. Posey and S. M. Woodward. [b] These values to be multiplied by 10^{-6}.

In order to solve this problem with the graphical device suggested by Messrs. Woodward and Posey, the values of $f(y)$ are plotted against the depths in Fig. 15. The area (A) under the curve is equal to the distance from the control section to the station where the depth is 4.00 ft; the area (B) equals

[2a] Tables for the values of the factors u and j are available for the cases of vertical side walls and side walls having a slope of 1 on 1 and 1 on 1.5 with bottom widths ranging from 5 ft to 50 ft in 5-ft intervals and depths from 1 ft to 10 ft in 0.5-ft intervals. The cost of these tables is $1.00.

the distance from where $y = 4.00$ to where $y = 3.50$ ft, etc. The water surface profile determined by this method is superimposed on Fig. 14 for comparison.

For finding the water surface by use of the Husted method, Eqs. 51*a* and 51*b* can again be advantageous. As is customary in the step methods, the values of the variables will be averaged for each length of channel. The factors u and j in Eq. 51*b*, therefore, will be taken as the averages of the functions at the two stations that define the increment. The results summarized in Table 13(*b*) were obtained by applying Eq. 51*a* in finite terms.

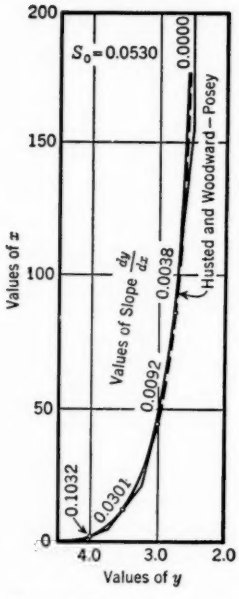

FIG. 14.—DETERMINATION OF WATER SURFACE SLOPE, EXAMPLE 3

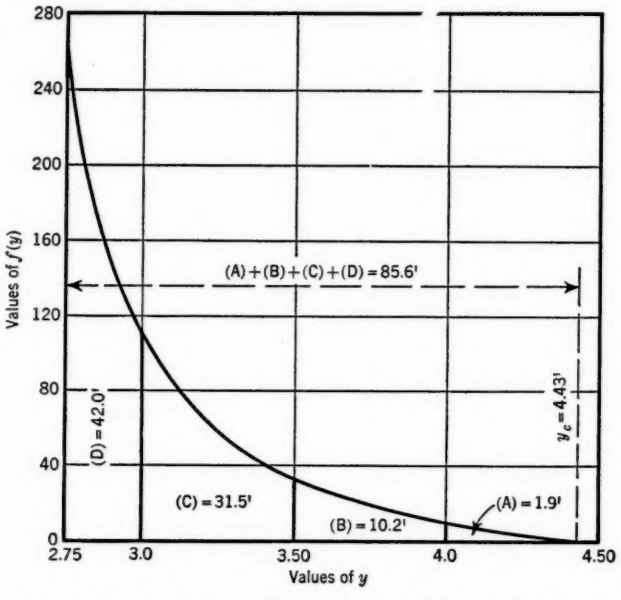

FIG. 15.—SOLUTION BY THE WOODWARD–POSEY METHOD, EXAMPLE 3

The lengths shown in Table 13(*b*) are seen to be practically the same as those in Fig. 15, the latter having been computed as suggested by Messrs. Woodward and Posey.

This typical problem has thus been solved by the Husted method, by the Woodward-Posey graphical procedure, and by direct plotting of the water surface slopes. In each of these three approaches, the general depth functions u and j have been used to reduce the labor of computation. The close conformity of the resulting water profiles, illustrated in Fig. 14, indicates that any of these solutions provides acceptable accuracy. Whether one or another method will best solve a given case will depend upon the length of the profile and upon the facilities available.

These closing statements serve to show that Mr. Stevens and the writer are not widely separated in their lines of attack. The two methods stem from the

same basic relationships and produce practically identical results. In each case the calculations are started from a known depth, and the depth functions for each increment are substituted into a formula such as Eq. 12. The effects of velocity distribution and eddy loss, if predictable, can be taken into account with equal ease by either method.

Mr. Stevens' distaste for lengthy formulas and laborious calculations is shared by the writer and by the multitude. The merits of the suggestions offered in this paper and its discussions will be found where there is a saving in the time spent by the designer.

Messrs. Bishay and Babbitt have contributed an interesting comparison of several methods by use of a single example involving a relatively long water surface curve. They have shown the writer's procedure to require probably the least volume of numerical calculation; but, at the same time, they indicate an appreciable deviation in the computed length along the channel for a given depth. This type of comparison is misleading because it considers only relative horizontal distances instead of relative depths. By this method it would be possible to show that two curves approaching near the normal depth could differ in length by a percentage approaching infinity, even though the actual depths at a given station were nearly identical. This is a ludicrous example, of course, but it should illustrate the uncertainties of such a comparison.

The practical engineer wants to know how deep the stream is at a point 11,000 ft down the channel, not how far it is to where the depth is 7.0 ft. He wants to know how high to build his sidewalls and how much freeboard he will need. The percentage deviation of depths computed here would be well within the limits usually required by the designer.

As Mr. Nelidov has so ably stated, the only correct solution is by integration. However, common methods of integrating the water surface equation involve their own approximations and give accuracy which is not always adequate compensation for the additional labor required. In most practical cases, ease of calculation is more attractive than laborious and unnecessary refinement.

In the introduction of the method of direct plotting of the water surface curve, the tangents were arbitrarily made to intersect at the midpoints of the depth increments in order to attain uniform results for illustration. It was stated that this procedure can be modified in practice to give more accuracy in that part of the curve where the slope is varying rapidly. In problems solved by the writer through use of the unmodified procedure, the discrepancy in depth has not been greater than 4%, the average being about 2%. If more accuracy is required, smaller depth increments can be taken.

Mr. Pietrkowski states that direct plotting is restricted to a special shape of cross section. The method is in fact free of any such restriction. Although the final equations presented here are limited to trapezoidal channels, the potential applications of the basic idea are manifold.

The thought-provoking international responses to this paper have enhanced its value immeasurably.

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